Quiz 10 Summary

#1) Is it possible for

$$\nabla \times \vec{F} = \langle y, -x, z \rangle ?$$

Two very important identities (both directly proven using Clairaut's)

curl (gradient (any* scalar fn)) = $\vec{0}$

$\nabla \times \nabla f$

– and –

div (curl (any* vec. field)) = 0

$\nabla \cdot (\nabla \times \vec{F})$

*nice (i.e. sufficiently differentiable)

Sol)

$$0 = \nabla \cdot (\nabla \times \vec{F}) = \nabla \cdot \langle y, -x, z \rangle = 0 + 0 + 1$$
$$= 1$$

Contradiction.

So $\vec{F}$ cannot exist. $\square$

⚠ What if the question was "is it possible for

$$\nabla \times \vec{F} = \langle y, -y, z \rangle \text{"} ?$$

$\nabla \cdot \langle y, -y, z \rangle = 0$

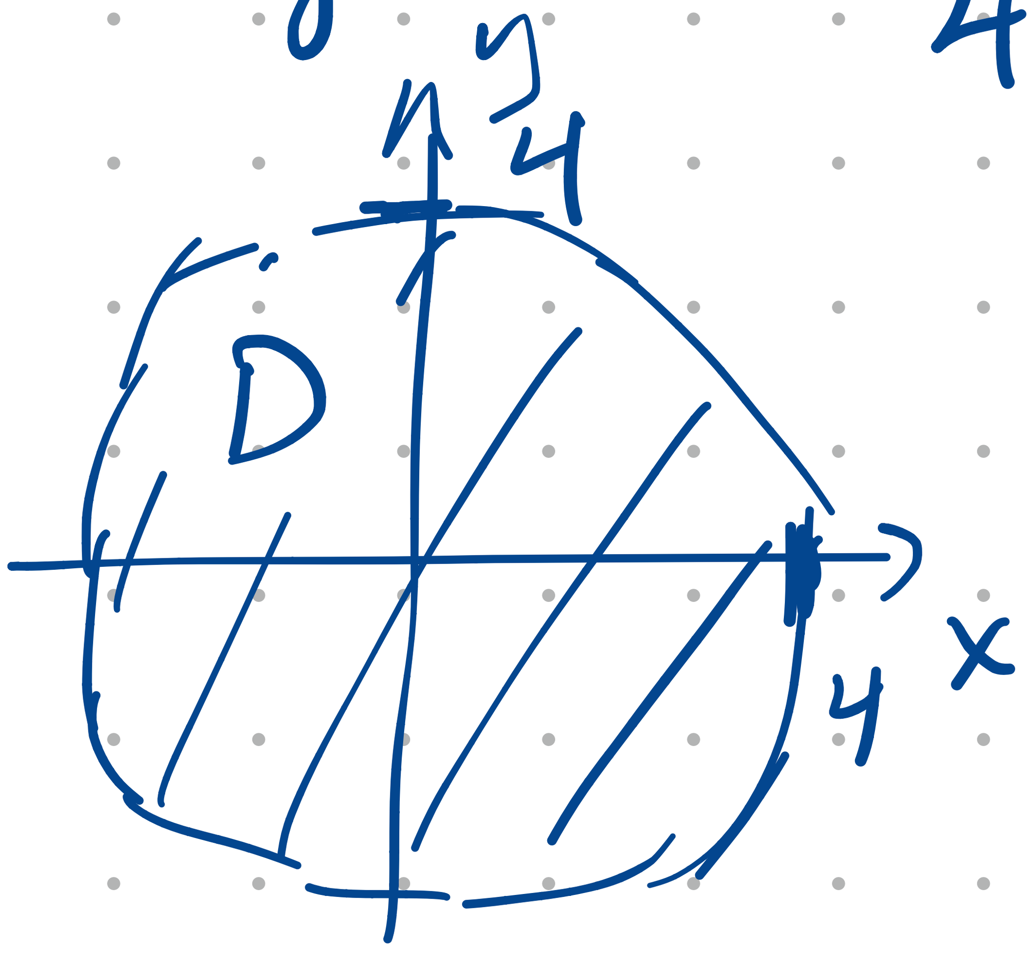
so the preceeding argument doesn't work to conclude the answer is "no".

(Actually the answer is "yes" in this case, but that doesn't follow from what I've written so far)

#3) <u>Method 1</u> (Calculus)

$$\vec{r}(x,y) = \langle x, y, \tfrac{3}{4}\sqrt{x^2+y^2} \rangle$$

parameter region $\quad \frac{3}{4}\sqrt{x^2+y^2} \le 3$

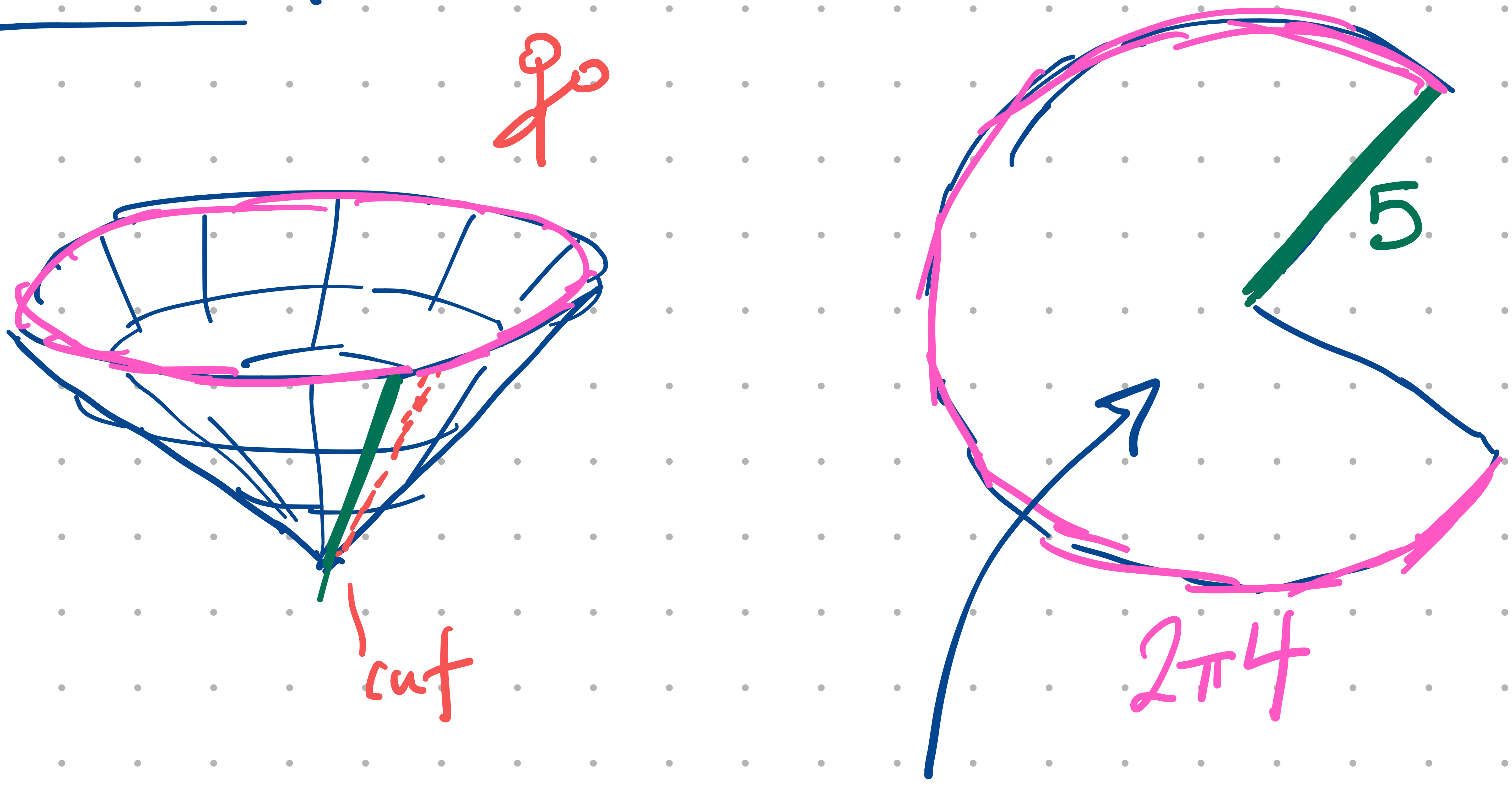


Then compute

$$\iint_D |\vec{r}_x \times \vec{r}_y| \, dx\,dy$$

(the $|\vec{r}_x \times \vec{r}_y|$ turns out to just be a constant, so the integral is easy).

Method 2 (w/o Calculus) ⚠ only works for cone



From here, can compute area of

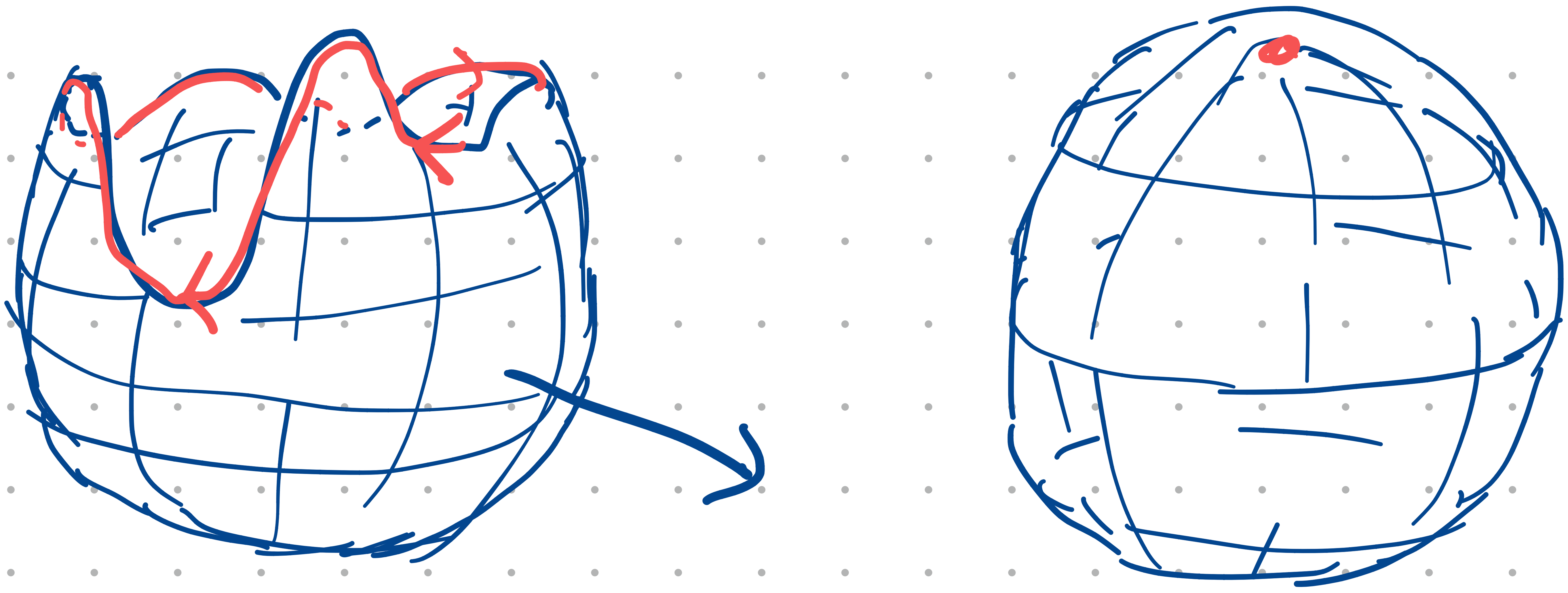
#2) S: $x^2+y^2+z^2=1$ outwards

$$\iint_S \left(\nabla\times\langle -y,x,z\rangle\right)\cdot d\vec{S}$$

Solution 1: Stokes (hinted @ by flux of a curl in problem)

$$0 = \oint \langle -y,x,z\rangle\cdot d\vec{r} = \iint_S \left(\nabla\times\langle -y,x,z\rangle\right)\cdot d\vec{S}$$

boundary of S



but the boundary of S is a single point

(or "nothing" as people would just say S has no boundary).

Solution 2: Divergence Thm

$$\oiint_S (\nabla \times \langle -y, x, z \rangle) \cdot d\vec{S} = \iiint \nabla \cdot (\nabla \times \langle -y, x, z \rangle)\, dV$$

region enclosed by S

i.e. $x^2+y^2+z^2 \leq 1$

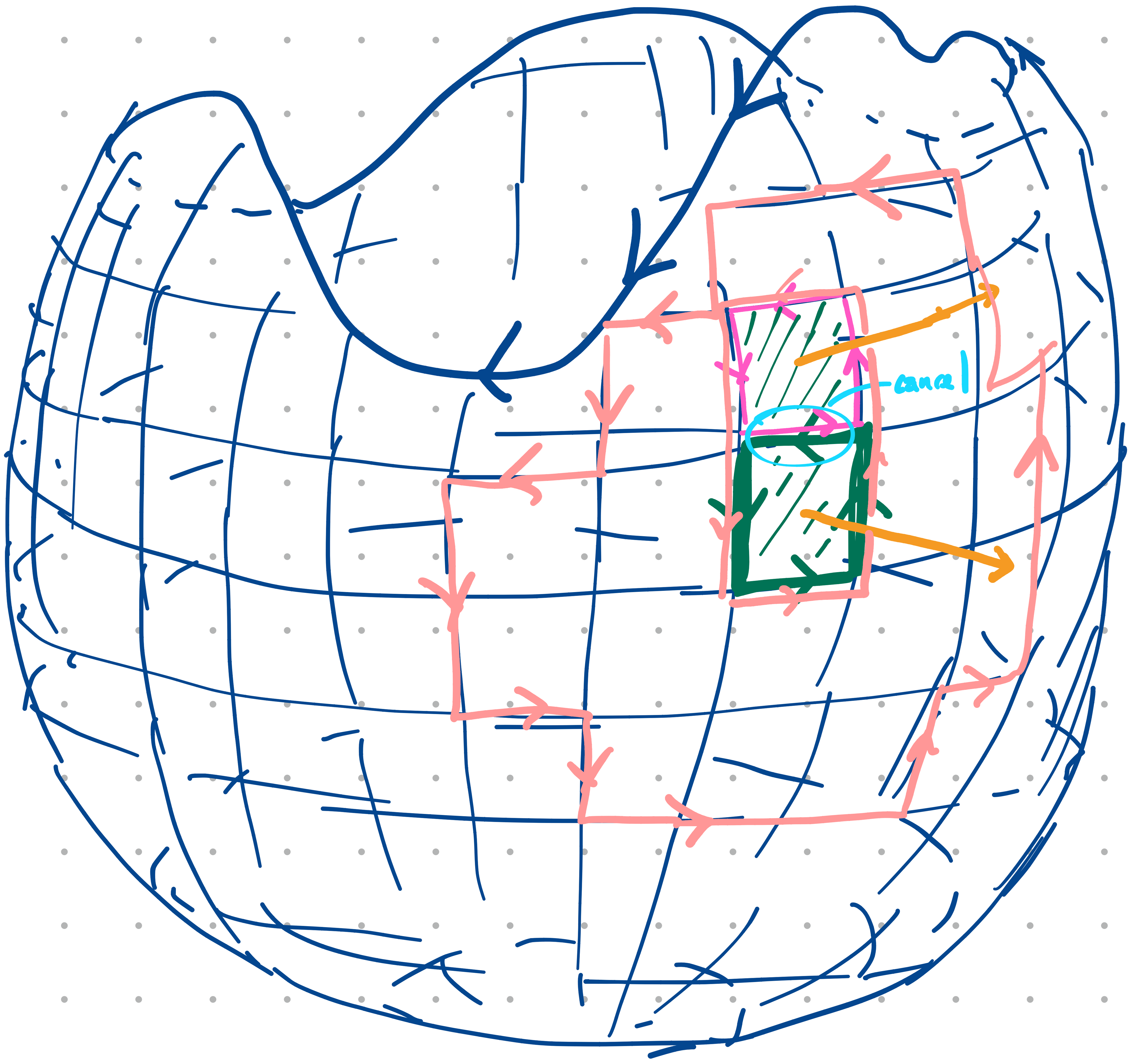
$$= \iiint_{x^2+y^2+z^2 \leq 1} 0 \, dV = 0.$$

⚠ If the problem were instead

$$\iint_S \left(\nabla \times \left\langle \frac{-y}{x^2+y^2+z^2}, \frac{x}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \right\rangle\right) \cdot d\vec{S}$$

the answer would still be 0 (e.g. by Sol. 1)

but Sol. 2 is no longer valid.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